Estimation of Tropical Cyclone Wind Using a Modified Jelesnianski’s Pressure Model

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Abstract

In this paper, a parametric wind model for estimation of the gradient wind speed of tropical cyclone is developed. The model is based on the gradient wind model and the Jelesnianki’s pressure model. The model parameters are the pressure of the storm center, the pressure of environment and the radius of maximum wind speed. The case study is typhoon Vamei (2001). Results from the model show that the gradient wind speed of tropical cyclone has good accuracy.

Keywords: Wind Model, Tropical Cyclone, Jelesnianki’s Pressure Model, Typhoon Vamei

Introduction

Cyclones, typhoon and hurricane are the names given in different parts of the world to tropical storms. Each year, many areas of the world are struck by tropical cyclones. Tropical cyclones can cause immense damage loss in life and property. C.Raktham¹ studied numerical simulation of associated atmospheric dynamic processes and tracking of the tropical storm Vicente during September 14-18, 2005 by employing the Weather Research Forecasting Model (WRF). W. Wannawong² studied tropical cyclone wind wave, storm surge and current characteristics in the South China Sea. In Thailand there is no mathematical model for the purpose of tropical cyclone wind estimation. Thus it is necessary to develop a wind model for Thailand. In this paper, a parametric wind model of tropical cyclone is developed.

The severity of a tropical cyclone is described in terms of categories ranging from 1 (weakest) to 5 (strongest) related to the zone of maximum wind gusts as shown in Table 1.

<table>
<thead>
<tr>
<th>Category</th>
<th>Maximum wind Speed (km/h)</th>
<th>Central Pressure (hPa)</th>
<th>Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>63-88</td>
<td>&gt;985</td>
<td>Negligible house damage.</td>
</tr>
<tr>
<td>2</td>
<td>89-117</td>
<td>985-970</td>
<td>Minor house damage. Significant damage to signs, trees.</td>
</tr>
<tr>
<td>3</td>
<td>118-159</td>
<td>970-955</td>
<td>Some roof and structural damage.</td>
</tr>
<tr>
<td>4</td>
<td>160-199</td>
<td>955-930</td>
<td>Significant roofing loss structural damage.</td>
</tr>
<tr>
<td>5</td>
<td>&gt;200</td>
<td>&lt;930</td>
<td>Extremely dangerous with widespread destruction.</td>
</tr>
</tbody>
</table>

Methodology

Tropical Cyclone Wind Model

The structure of the wind field in a tropical cyclone is examined from the momentum equation expressed in terms of cylindrical coordinate system⁴.

\[
\frac{V^2}{r} + fV - \frac{1}{\rho} \frac{\partial P}{\partial r} = 0 \tag{1}
\]

Equation (1) is the gradient wind equation with the centrifugal force \(\frac{V^2}{r}\), the Coriolis force \(fV\) and the pressure gradient force \(-\frac{1}{\rho} \frac{\partial P}{\partial r}\).

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The gradient wind speed is obtained by solving Eq. (1) for $V$ to yield

$$V = \left( \frac{fr^2}{2} + \frac{r}{2} \frac{\partial P}{\partial r} - \frac{fr}{2} \right)^{1/2}$$  \tag{2}

where $f$ is the Coriolis parameter, $r$ is the radial distance from the center of the storm, $P(r)$ is the pressure at the distance $r$ and $\rho$ is the air density.

The pressure is computed from Jelesnianski’s model [5]

$$P = \frac{1}{\gamma} \left( \frac{V_{\text{max}}}{r} \right)^2 \left( \frac{r}{R} \right)^3 + P_c ; 0 \leq r < R$$ \tag{3}

$$P = -\rho \left( \frac{V_{\text{max}}}{r} \right)^2 \left( \frac{R}{r} \right) + P_a ; r \geq R$$ \tag{4}

where $\gamma = 2V_{\text{max}} \frac{\rho}{3} (P_a - P)$ and $V_{\text{max}}$ is the maximum wind speed, $R$ is the radius of maximum wind, $r$ is the radial distance from the center of the storm, $\rho$ is the air density, $P_a$ is the pressure at the environment, $P_c$ is the pressure at the storm center. Equation (3) represents the pressure within the radius of maximum wind and equation (4) is for the pressure outside the radius of the maximum wind.

The pressure gradient from differentiation of Eq. (3) and Eq. (4) are

$$\frac{\partial P}{\partial r} = \rho \left( \frac{V_{\text{max}}}{\gamma} \right)^2 \left( \frac{r^2}{R^2} \right) ; 0 \leq r < R$$  \tag{5}

$$\frac{\partial P}{\partial r} = \rho \left( \frac{V_{\text{max}}}{\gamma} \right)^2 \left( \frac{R}{r^2} \right) ; r \geq R$$  \tag{6}

Substitute Eq. (5) and Eq. (6) into Eq. (2)

$$V = \left( \frac{fr^2}{2} + \frac{r}{2} \frac{\partial P}{\partial r} - \frac{fr}{2} \right)^{1/2} ; 0 \leq r < R$$ \tag{7}

$$V = \left( \frac{fr^2}{2} + \frac{r}{2} \frac{V_{\text{max}}}{\gamma} \right)^{1/2} \left( \frac{r^2}{R^2} \right) - \frac{fr}{2} ; r \geq R$$ \tag{8}

In this research Eq. (7) and Eq. (8) are used to estimate the gradient wind speed of tropical cyclone.

**Steps to Solve the Model Equations**

**Step 1:** Input pressure at the storm center ($P_c$), the pressure of the environment ($P_a$), the distance from the center of the storm ($r$), the air density and the maximum wind speed.

**Step 2:** Calculate the parameter $\gamma$ from

$$\gamma = 2V_{\text{max}} \frac{\rho}{3} (P_a - P_c)$$

**Step 3:** Substitute the parameter $\gamma$ from Step 2 into Eq. (5) and Eq. (6) to calculate the pressure gradient.

**Step 4:** Substitute the pressure gradient from Step 3 into Eq. (7) and Eq. (8) to calculate the gradient wind speed.

These steps are shown in Figure 1.

![Figure 1](1)

**Figure 1** Flow chart for calculation of the gradient wind speed.
**Experiment Case**

In this research, the study case is typhoon Vamei, because typhoon Vamei caused strong winds and heavy rainfall. Observation data of typhoon Vamei are shown in Table II and the track is shown in Figure 2.

<table>
<thead>
<tr>
<th>Name</th>
<th>Case</th>
<th>Date/Time</th>
<th>Position</th>
<th>Maximum Wind Speed (m/s)</th>
<th>Minimum Pressure (hPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAMEI</td>
<td>1</td>
<td>27/12/2001/00</td>
<td>1.5</td>
<td>105.1</td>
<td>33.44</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>27/12/2001/06</td>
<td>1.5</td>
<td>104.4</td>
<td>33.44</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>27/12/2001/12</td>
<td>1.6</td>
<td>103.7</td>
<td>23.15</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>27/12/2001/18</td>
<td>1.6</td>
<td>102.9</td>
<td>20.58</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>28/12/2001/00</td>
<td>1.7</td>
<td>102.2</td>
<td>15.43</td>
</tr>
</tbody>
</table>

**Results and Discussion**

The gradient wind speeds from this model are shown in Figure 3.
The maximum wind speeds from the model are shown in Table 3. The mean absolute error between observed maximum wind speed and the maximum wind speed from the model is 1.94. Results from the model show that the gradient wind speed of tropical cyclone has good accuracy (error < 20%).

Table 3 The maximum wind speed from the model compared with the maximum wind speed from observed data.

<table>
<thead>
<tr>
<th>Name</th>
<th>Case</th>
<th>Observed Maximum Wind Speed (m/s)</th>
<th>Maximum Wind Speed from Model (m/s)</th>
<th>Absolute Error (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAMEI</td>
<td>1</td>
<td>33.44</td>
<td>33.31</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>33.44</td>
<td>33.31</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>23.15</td>
<td>26.96</td>
<td>3.81</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>20.58</td>
<td>24.34</td>
<td>3.76</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>15.43</td>
<td>17.29</td>
<td>1.86</td>
</tr>
<tr>
<td>Mean Absolute Error</td>
<td>1.94</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conclusion
A tropical cyclone wind model is developed based on the gradient and Jesnianki’s pressure models. The parameters in this model are the pressure of storm center, the pressure of the environment and the radius of maximum wind. Typhoon Vamei (2001) is used as the study case. Results from the application of the model show that the wind speeds from the model are good enough for practical use.

For recommendation, estimated wind fields in tropical cyclones from various models should be compared.

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References