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Transmuted Generalized Inverted Exponential Distribution with Application to Reliability Data

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Abstract

This paper investigates the potential usefulness of the three parameter transmuted generalized inverted exponential distribution for modelling upside-down bathtub shaped instantaneous failure rates. Some structural properties of this distribution are discussed. Explicit expressions are derived, such as quantile, Rényi entropy and the r^{th} moment of order statistics. The method of maximum likelihood is used for estimating the model parameters and illustrate this model through an application to failure data.

Keywords: Reliability functions, order statistics, entropy, simulation, maximum likelihood estimation.

1. Introduction

The exponential family of distributions are popular for modelling lifetime data in biomedical sciences, reliability engineering, astronomy, marine sciences, medicine, psychology, agriculture, botany, zoology, reliability and life testing. The exponential distribution is very popular for modelling constant instantaneous failure rates. For non-linear cases, the role of the negative exponential distribution is important for many areas of reliability engineering and failure data. The generalized inverted exponential distribution was first introduced and studied by Abouammoh and Alshingiti (2009) as a generalization of one parameter inverted exponential distribution, see Dey 2007. Dey et al. (2014) obtained the maximum likelihood estimates, and Bayes estimates under squared error loss function and discussed various structural properties with applications. The cumulative distribution function (cdf) of the generalized inverted exponential (GIE) distribution is given by

$$F(t) = 1 - \left\{1 - \exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}, \quad (1)$$

where θ is the scale parameter and ϕ is the shape parameter. Using the quadratic rank transmutation map studied by Shaw et al. (2007), we can obtain the three parameter TGIE distribution. A random variable T is said to have transmuted distribution if its cdf is given by

$$F(t) = (1+\lambda)G(t) - \lambda G(t)^2, \ |\lambda| \le 1, \quad (2)$$

where $G(t)$ is the cdf of the baseline model. For $\lambda = 0$, we have the distribution of the base random variable (Shaw et al. 2007). Elbatal (2014) proposed and studied some mathematical properties of the transmuted generalized inverted exponential (TGIE) distribution and discussed some mathematical properties of this model, which includes the moments, moment generating function and method of maximum likelihood. This paper investigates the potential usefulness of the three parameter transmuted generalized inverted exponential (TGIE) distribution with an application to reliability data. This research will discuss the quantile analysis, Rényi entropy, the k^{th} fractional moment of the r^{th} order statistic. Remarkably, extensive work has been done on the transmuted family of distributions, such as Gokarna et al. (2011) proposed and studied the transmuted Weibull distribution with application. Khan and King (2013a, 2013b) developed the transmuted modified Weibull and transmuted generalized inverse Weibull distributions. Recently, Khan et al. (2014a, 2014b) proposed the transmuted inverse Weibull distribution and discussed various structural properties with applications. More recently Khan and King (2015) proposed the transmuted modified Inverse Rayleigh distribution and discussed some of its mathematical properties with application. Merovci (2013) studied the transmuted Rayleigh distribution with application to nicotine in cigarettes data. This article outlines the usefulness of the TGIE distribution with some mathematical properties such as quantile function, Rényi entropy and the k^{th} fractional moment of the r^{th} order statistics. This paper also presents the graphical analysis of the TGIE distribution.

The article is organized as follows. Section 2 presents the analytical shapes of the probability density, reliability and hazard functions of the TGIE model. Section 3, discusses the feature of the quantile function. Entropy is derived in Section 4. In Section 5, we derive the k^{th} moment of the r^{th} order statistics. Maximum likelihood estimates (MLEs) of the unknown parameters and the asymptotic confidence intervals of the TGIE models are discussed in Section 6. Section 7 illustrates the usefulness of the TGIE model by means of an application to real data. Concluding remarks are addressed in Section 8.

2. Transmuted Generalized Inverted Exponential Distribution

A random variable T is said to have TGIE distribution with parameters θ, $\phi > 0$, $|\lambda| \leq 1$ and $t > 0.$ The probability density function is given by

$$f(t) = \frac{\phi\theta}{t^2}\exp\left(-\frac{\theta}{t}\right)\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi-1}\left\{1-\lambda+2\lambda\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}. \tag{3}$$

The cumulative distribution function (cdf), reliability function (rf) and hazard function (hf) corresponding to (3) are given by

$$F(t) = \left\{1-\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}\left\{1+\lambda\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}, \tag{4}$$

$$R(t) = 1-\left\{1-\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}\left\{1+\lambda\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}, \tag{5}$$

and

$$h(t) = \frac{\frac{\phi\theta}{t^2}\exp\left(-\frac{\theta}{t}\right)\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi-1}\left\{1-\lambda+2\lambda\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}}{1-\left\{1-\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}\left\{1+\lambda\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}}, \tag{6}$$

where θ and ϕ are the scale and shape parameters, λ is the transmuted parameter representing the different patterns of the TGIE distribution. If T is a random variable having the transmuted generalized inverted exponential (TGIE) distribution with pdf (3), denoted as $T \sim TGIE(t;\phi,\theta,\lambda)$.

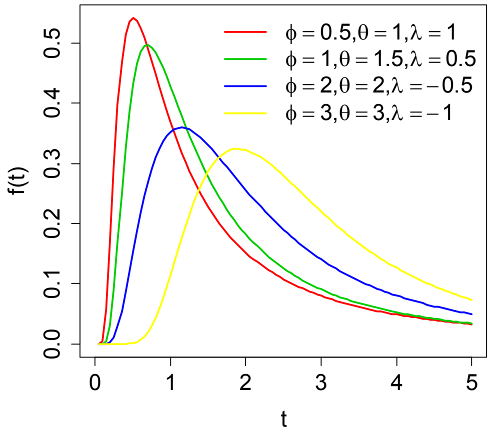

Figure 1 Plot of the TGIE density function

Figure 1 shows the diverse shape of the TGIE pdf with different choice of parameters. The transmuted inverse exponential (TIE) distribution proposed by Oguntunde and Adejumo (2015), is the special case of the TGIE distribution for the shape parameter $\phi = 1$. The generalized inverted exponential (GIE) distribution was proposed by Abouammoh and Alshingiti (2009), is the special case of the TGIE model when the transmuting parameter $\lambda = 0$. If $\phi = 1$, in addition to $\lambda = 0$, the TGIE distribution reduces to the inverse exponential (IE) distribution.

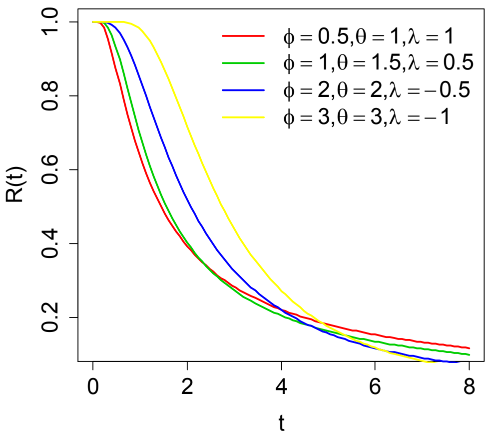

Figure 2 Plot of the TGIE reliability function

Figure 2 illustrates the reliability pattern of the TGIE distribution with different choice of parameters. The reversed hazard function for the TGIE distribution also known as failure rate denoted by $r(t)$ defined as

$$r(t)=\frac{\frac{\phi\theta}{t^2}\exp\left(-\frac{\theta}{t}\right)\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi-1}\left\{1-\lambda+2\lambda\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}}{\left\{1-\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}\left\{1+\lambda\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}}. \tag{7}$$

By definition $H(t)=\int_0^t h(t)dt$, the cumulative hazard function of the TGIE distribution corresponding to (3) is given by

$$H(t)=-\ln\left\{1-\left\{1-\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}\left\{1+\lambda\left\{1-\exp\left(-\frac{\theta}{t}\right)\right\}^{\phi}\right\}\right\}. \tag{8}$$

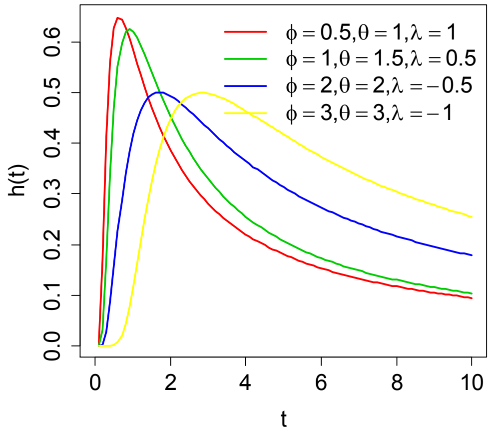

Figure 3 Plot of the TGIE hazard function

Figure 3 illustrates the hazard rates of the TGIE model. These failure rates are defined with different choices of parameters. The TGIE distribution has strictly increasing and decreasing patterns of hazard rates.

3. Properties of the Quantile Function

The quantile t_q of the $\text{TGIE}(t;\phi,\theta,\lambda)$ is the real solution of the following equation

$$t_q=-\theta\left[\ln\left\{1-\left[1-\frac{(1+\lambda)-\sqrt{(1+\lambda)^2-4\lambda q}}{2\lambda}\right]^{\frac{1}{\phi}}\right\}\right]^{-1}. \tag{9}$$

To substitute $q=0.5$ in (9), we obtain the median of the TGIE distribution. One can use (9), to derive the following special cases:

1. The q -th quantile of the transmuted inverse exponential (TIE) distribution by setting the shape parameter $\phi=1$ as

$$t_q=-\theta\left[\ln\left\{\frac{(1+\lambda)-\sqrt{(1+\lambda)^2-4\lambda q}}{2\lambda}\right\}\right]^{-1}.$$

2. The q-th quantile of the generalized inverted exponential (GIE) distribution by setting the transmuting parameter $\lambda = 0$ as

$$t_q = -\theta\left[\ln\left\{1-[1-q]^{\frac{1}{\phi}}\right\}\right]^{-1}.$$

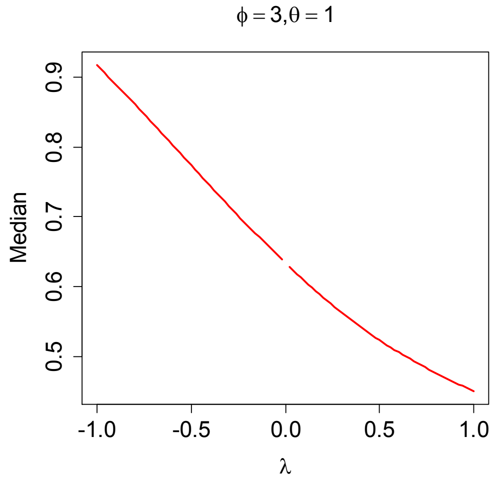

Figure 4 Plot of the TGIE Median

Figure 4 shows the median life to illustrate the effect of transmuted parameter. Figure 5 shows the coefficient of quartile deviation as a function of the transmuting parameter λ. To illustrate the effect of transmuting parameter on skewness and kurtosis, we consider the measure based on quantiles. The skewness and kurtosis measures can now be calculated from the quantiles using Bowley (B) and Moors (M) well known relationships. Graphical representations of the Bowley (B) skewness and Moors (M) kurtosis when $\phi = 3$ and $\theta = 1$, as a function of the transmuting parameter λ are illustrated in Figures 6 and 7, respectively.

The Bowley's skewness and Moors kurtosis are given by

$$B = \frac{Q\left(\frac{3}{4}\right)+Q\left(\frac{1}{4}\right)-Q\left(\frac{2}{4}\right)}{Q\left(\frac{3}{4}\right)-Q\left(\frac{1}{4}\right)}, \tag{10}$$

and

$$M = \frac{Q\left(\frac{3}{8}\right)+Q\left(\frac{1}{8}\right)+Q\left(\frac{7}{8}\right)-Q\left(\frac{5}{8}\right)}{Q\left(\frac{6}{8}\right)-Q\left(\frac{2}{8}\right)}. \tag{11}$$

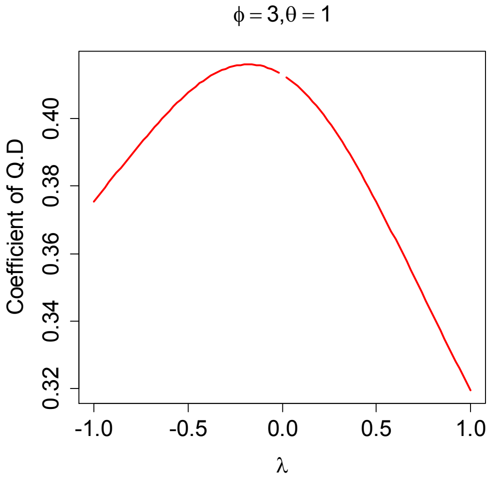

Figure 5 Plot of the TGIE λ vs. coefficient of Q.D.

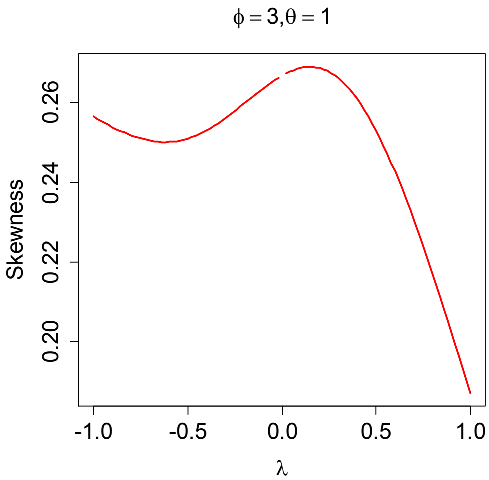

Figure 6 Plot of the TGIE λ vs. Bowley (B) skewness

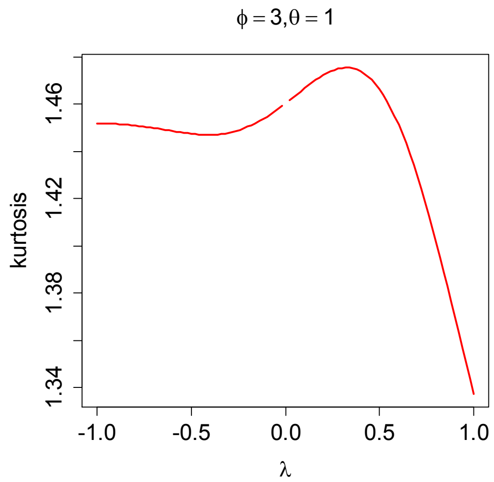

Figure 7 Plot of the TGIE λ vs. Moors (M) kurtosis

4. Entropy

The entropy of a random variable T with density $f(t)$ is a measure of variation of the uncertainty. A large value of entropy indicates the greater uncertainty in the data. The Rényi entropy is defined as

$$I_R(\rho)=\frac{1}{1-\rho}\log\left\{\int f(t)^{\rho}\,dt\right\}, \tag{12}$$

where $\rho>0$ and $\rho\neq 1$. The integral in $I_R(\rho)$ of the TGIE$(t;\phi,\theta,\lambda)$ can be defined as

$$\int_0^{\infty} f(t)^{\rho}\,dt=\int_0^{\infty}\frac{\phi^{\rho}\theta^{\rho}}{t^{2\rho}}\exp\left(-\rho\frac{\theta}{t}\right)\left[1-\exp\left(-\frac{\theta}{t}\right)\right]^{\rho(\phi-1)}\left\{1-\lambda+2\lambda\left[1-\exp\left(-\frac{\theta}{t}\right)\right]^{\phi}\right\}^{\rho}dt$$

$$=\sum_{k=0}^{\infty}W_{\phi,\theta,\rho,\lambda,K}\int_0^{\infty}t^{-2\rho}\left[1-\exp\left(-\frac{\theta}{t}\right)\right]^{\rho(\phi-1)+k\phi}dt,$$

where $W_{\phi,\theta,\rho,\lambda,K}=\phi^{\rho}\theta^{\rho}\binom{\rho}{k}\left(\frac{2\lambda}{1-\lambda}\right)^{k}(1-\lambda)^{\rho}$,

$$\int_0^{\infty} f(t)^{\rho}\,dt=\sum_{k,l=0}^{\infty}W_{\phi,\theta,\rho,\lambda,K}\binom{\rho(\phi-1)+\phi k}{l}(-1)^{l}\theta^{-2\rho+1}l^{-2\rho+3}\Gamma(2\rho-1). \tag{13}$$

Using equations (12) and (13), we obtain the Rényi entropy as

$$I_R(\rho)=\frac{\rho}{1-\rho}\log(\phi)+\frac{\rho}{1-\rho}\log(\theta)+\frac{\rho}{1-\rho}\log(1-\lambda)$$
$$+\frac{1}{1-\rho}\log\left\{\sum_{k,l=0}^{\infty}\binom{\rho}{k}\binom{\rho(\phi-1)+\phi k}{l}(-1)^{l}\left(\frac{2\lambda}{1-\lambda}\right)\theta^{-2\rho+1}l^{-2\rho+3}\Gamma(2\rho-1)\right\}. \tag{14}$$

Table 1 Quartile measures of the TGIE distribution for the parameter vector Θ

θ	ϕ	λ	Estimates					
			Q_1	Q_2	Q_3	IQR	QD	CQD
0.5	1	-1.0	1.6609	3.3219	8.0039	6.3429	3.1714	0.6562
		-0.5	1.1455	2.3924	5.9055	4.7600	2.3800	0.6750
		0.5	0.6651	1.1962	2.5261	1.8610	0.9305	0.5831
		1.0	0.5727	0.9375	1.6609	1.0882	0.5441	0.4871
1	1	-1.0	3.3219	6.6438	16.007	12.685	6.3429	0.6562
		-0.5	2.2910	4.7849	11.811	9.5201	4.7600	0.6750
		0.5	1.3302	2.3924	5.0523	3.7220	1.8610	0.5831
		1.0	1.1455	1.8751	3.3219	2.1764	1.0882	0.4871
1	2	-1.0	1.8751	2.9553	5.0523	3.1771	1.5885	0.4586
		-0.5	1.4474	2.3924	4.2155	2.7680	1.3840	0.4887
		0.5	0.9688	1.4927	2.4789	1.5100	0.7550	0.4379
		1.0	0.8630	1.2526	1.8751	1.0120	0.5060	0.3696
2	3	-1.0	2.9175	4.2212	6.4245	3.5069	1.7534	0.3753
		-0.5	2.3502	3.5615	5.5845	3.2343	1.6171	0.4076
		0.5	1.6649	2.4121	3.6653	2.0003	1.0001	0.3752
		1.0	1.5042	2.0786	2.9175	1.4133	0.7066	0.3196
2	4	-1.0	2.5052	3.4611	4.9578	2.4525	1.2262	0.3286
		-0.5	2.0664	2.9855	4.4017	2.3352	1.1676	0.3610
		0.5	1.5120	2.1150	3.0614	1.5493	0.7747	0.3387
		1.0	1.3774	1.8502	2.5052	1.1278	0.5639	0.2904
2	5	-1.0	2.2525	3.0210	4.1655	1.9130	0.9565	0.2980
		-0.5	1.8869	2.6429	3.7474	1.8604	0.9302	0.3301
		0.5	1.4109	1.9279	2.7038	1.2928	0.6464	0.3141
		1.0	1.2925	1.7033	2.2525	0.9599	0.4799	0.2707

5. Order Statistics

The pdf of the r^{th} order statistic $t_{(r)}$ of random sample $t_1, t_2, ..., t_n$ drawn from the TGIE distribution is given by

$$f_{r:n}(t) = \frac{n!}{(r-1)!(n-r)!}\left[F(t)\right]^{r-1}\left[1-F(t)\right]^{n-r} f(t). \tag{15}$$

Substituting (3) and (4) in (15), by setting $u_i = 1-\exp\left(-\frac{\theta}{t}\right)$, defined as

$$f_{r:n}(t) = n\binom{n-1}{r-1}\sum_{p=0}^{n-r}\binom{n-r}{p}(-1)^p\left\{\left[1-u^{\phi}\right]\left[1+\lambda u^{\phi}\right]\right\}^{r+p-1}\frac{\phi\theta}{t^2}\exp\left(-\frac{\theta}{t}\right)u^{\phi-1}\left(1-\lambda+2\lambda u^{\phi}\right)$$

$$= n\binom{n-1}{r-1}\sum_{p=0}^{n-r}\sum_{q,s=0}^{\infty} w_{p,q,s,\lambda} J\left(\theta,\phi,u,\lambda\right), \tag{16}$$

where $w_{p,q,s,\lambda} = \binom{n-r}{p}\binom{r+p-1}{q}\binom{r+p-1}{s}(-1)^{p+q+s}\lambda^s$ and

$J(\theta,\phi,u,\lambda) = \phi\theta\left[\ln\left(\frac{1}{1-u}\right)\right]^2 (1-u)u^{\phi(q+s+1)-1}\left(1-\lambda+2\lambda u^{\phi}\right).$

Using (16), the k^{th} fractional moment of the r^{th} order statistic of $t_{(r)}$ is given by

$$\mu_k^{(r:n)} = n\binom{n-1}{r-1}\sum_{p=0}^{n-r}\sum_{q,s=0}^{\infty} w_{p,q,s,\lambda}\left[(1-\lambda)T_1(\theta,\phi,q,s,v,k)+2\lambda T_2(\theta,\phi,q,s,v,k)\right], \tag{17}$$

where $T_g(\theta,\phi,q,s,v,k) = \sum_{v=0}^{\infty}\binom{\phi(q+s+g)-1}{v}\phi(-1)^v\theta^k(v+1)^{k-1}\Gamma(1-k)$, $g=1,2$.

6. Maximum Likelihood Estimation

This section examines the method of maximum likelihood and also provides expressions for the associated Fisher information matrix. Consider the random samples $t_1,t_2,...,t_n$ consisting of n observations from the TGIE distribution having probability density function. The log-likelihood function $L' = \ln L$ of (3) is given by

$$L' = n\ln\phi + n\ln\theta + \sum_{i=1}^{n}\ln\left(\frac{1}{t_i^2}\right) + \theta\sum_{i=1}^{n}\frac{1}{t_i} + (\phi+1)\sum_{i=1}^{n}\ln\left[1-\exp\left(-\frac{\theta}{t_i}\right)\right]$$
$$+\sum_{i=1}^{n}\ln\left\{1-\lambda+2\lambda\left[1-\exp\left(-\frac{\theta}{t_i}\right)\right]^{\phi}\right\}. \tag{18}$$

By differentiating (18) with respect to θ, ϕ and λ then equating it to zero, we obtain the estimating equations as follows.

$$\frac{\partial L'}{\partial\theta} = \frac{n}{\theta} - \sum_{i=1}^{n}\frac{1}{t_i} + (\phi+1)\sum_{i=1}^{n}\frac{\frac{1}{t_i}\exp\left(-\frac{\theta}{t_i}\right)}{1-\exp\left(-\frac{\theta}{t_i}\right)} + \sum_{i=1}^{n}\frac{2\lambda\left[1-\exp\left(-\frac{\theta}{t_i}\right)\right]^{\phi-1}\frac{1}{t_i}\exp\left(-\frac{\theta}{t_i}\right)}{1-\lambda+2\lambda\left[1-\exp\left(-\frac{\theta}{t_i}\right)\right]^{\phi}} = 0, \tag{19}$$

$$\frac{\partial L'}{\partial\phi} = \frac{n}{\phi} - \sum_{i=1}^{n}\ln\left[1-\exp\left(-\frac{\theta}{t_i}\right)\right] - \sum_{i=1}^{n}\frac{\left[1-\exp\left(-\frac{\theta}{t_i}\right)\right]^{\phi}\ln\left[1-\exp\left(-\frac{\theta}{t_i}\right)\right]}{(2\lambda)^{-1}\left\{1-\lambda+2\lambda\left[1-\exp\left(-\frac{\theta}{t_i}\right)\right]^{\phi}\right\}} = 0, \tag{20}$$

$$\frac{\partial L'}{\partial\lambda} = \sum_{i=1}^{n}\frac{-1+2\left[1-\exp\left(-\frac{\theta}{t_i}\right)\right]^{\phi}}{1-\lambda+2\lambda\left[1-\exp\left(-\frac{\theta}{t_i}\right)\right]^{\phi}} = 0. \tag{21}$$

By solving this nonlinear system of equations (19), (20) and (21), these solutions will yield the ML estimators $\hat{\theta}$, $\hat{\phi}$ and $\hat{\lambda}$. For estimating the parameters of the TGIE distribution, one can use the numerical iterative methods and statistical software can be used to solve them numerically such as R-Package (Adequacy Model), R language (2013).

Finally, simulation study performed to access the cost of additional transmuting parameter λ in (3) by using the method of maximum likelihood. We simulated samples of size n from the TGIE distribution for fixed choice of parameters $\theta = 1$, $\phi = 2$ and $\lambda = 0.5$. Table 2 shows the mean estimates, standard error, bias and mean square error of the TGIE model. The performances of most of the parameters estimates are quite good when the sample sizes are large.

Table 2 Mean, standard error, bias and MSE of the TGIE distribution

n	Parameter	Mean	S.E	Bias	MSE
50	θ	0.8922	0.1460	-0.1078	0.0329
	ϕ	1.9294	0.5715	-0.0706	0.3315
	λ	0.2562	0.4785	-0.2438	0.2884
100	θ	0.9872	0.1041	-0.0128	0.0110
	ϕ	1.9133	0.5035	-0.0867	0.2610
	λ	0.4578	0.3587	-0.0422	0.1304
200	θ	0.9837	0.0726	-0.0163	0.0055
	ϕ	1.7864	0.3950	-0.2136	0.2016
	λ	0.554	0.2773	0.054	0.0798
400	θ	0.9633	0.0492	-0.0367	0.0037
	ϕ	1.9216	0.2625	-0.0784	0.0750
	λ	0.5578	0.1624	0.0578	0.0297

7. Application

This section examines the real data analysis in order to assess the goodness-of-fit of the TGIE model in practice. The data set is obtained from Lawless (1982) represents the survival times for the 50 devices, the data are as follows.

0.1, 0.2, 1.0, 1.0, 1.0, 1.0, 1.0, 2.0, 3.0, 6.0, 7.0, 11.0, 12.0, 18.0, 18.0, 18.0, 18.0, 18.0, 21.0, 32.0, 36.0, 40.0, 45.0, 46.0, 47.0, 50.0, 55.0, 60.0, 63.0, 63.0, 67.0, 67.0, 67.0, 67.0, 72.0, 75.0, 79.0, 82.0, 82.0, 83.0, 84.0, 84.0, 84.0, 85.0, 85.0, 85.0, 85.0, 85.0, 86.0, 86.0

The transmuted generalized inverted exponential (TGIE), transmuted inverted exponential (TIE), Generalized inverted exponential (GIE) and inverted exponential (IE) distributions were fitted to this data set. The MLEs of the parameters with their standard errors and the AIC (Akaike information criteria) and AICC (Corrected Akaike information criteria) for the survival data is given in Table 3.

The values in Table 3, shows that the TGIE distribution provides a better fit than TIE, GIE and IE distributions. Because the TGIE distribution has the lowest AIC and AICC measures. Therefore, the TGIE distribution provides an adequate fit for the failure times of devices data. The likelihood ratio (LR) statistics for testing the hypothesis H_0: TIE $\times$ H_A: TGIE, H_0: GIE $\times$ H_A: TGIE and H_0: IE $\times$ H_A: TGIE are 65.8073 (p-value=4.97237E-16), 13.1848 (p-value=0.000282) and 93.4197 (p-value =4.23019E-22), respectively. Hence reject the null hypothesis in all cases in favour of the TGIE distribution. These results indicate that the TGIE is superior to using the TIE, GIE and IE distributions to fit the reliability data. This application suggests that the TGIE has the ability to fit right-skewed data set.

Table 3 MLEs of the parameters for devices data and the AIC, AICC measures

Distribution	Parameter Estimates			AIC	AICC
	$\hat{\phi}$	$\hat{\theta}$	$\hat{\lambda}$		
TGIE	0.3763 (0.0506)	0.6033 (0.2057)	-0.8447 (0.1249)	546.88	547.39
TIE	-	1.8430 (0.2842)	-0.8411 (0.1029)	610.68	610.94
GIE	0.2923 (0.0468)	0.8043 (0.2278)	-	558.06	558.32
IE	-	2.2592 (0.3195)	-	636.29	636.38

8. Concluding Remarks

This article discussed the performance of the transmuted generalized inverted exponential distribution and study some of its theoretical properties. The analytical shapes of density, reliability, hazard and quantile functions of the TGIE model are examined. Some mathematical properties were derived such as quantile function, Rényi entropy and the k^{th} fractional moment of the r^{th} order statistic. The TGIE distribution has increasing and decreasing failure rate patterns for lifetime data. The usefulness of the TGIE distribution is illustrated in an application to real data set and results shows the improved fit comparing with other three lifetime distributions.

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