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Modeling and Forecasting Volatility Series: with Reference to Gold Price

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Abstract

The recent global financial crisis has highlighted the need for financial institutions to find and implement of appropriate models for risk measurement. There was a particular interest of investors to increase their positions in the gold market as the risk in equity and bond markets was increasing. This study evaluates the effectiveness of various volatility models with respect to modeling and forecasting market risk in the gold future market. For this study, last trading price of gold futures are considered from January 1990 to June 2014 with 6,373 observations. The gold futures volatility is modeled and forecasted using GARCH-class models with long memory and fat-tail distributions, by considering ARMA model as the conditional returns. The results reveal that ARMA(1,1) model provides best results for the conditional returns. Among the linear and non-linear GARCH-class models, EGARCH and FIEGARCH models are provided best results for in-sampling forecasting. Moreover, EGARCH model gives bit of higher performance than FIEGARCH model under model diagnostic tests. After that, futures price volatilities of gold are forecasted using EGARCH and FIEGARCH models. Furthermore, it was found that long memory effect is significant. Forecasting accuracy of GARCH-class models are compared with different distributions of innovations. The results indicate that GARCH model with skew t-distribution outperform those with normal distribution. For speculations and noise traders in futures market, both linear and nonlinear models should be taken into account.

Keywords: Modeling, forecasting, gold futures, GARCH-class models.

1. Introduction

Given the rapid growth in financial markets and the current development of new and more complex financial instruments, there is an ever-growing need for theoretical and empirical knowledge of the volatility in financial time series. Therefore, modeling, analyzing, and forecasting volatility has been the subject of widespread research among academics and practitioners over the last decades. One complicated feature is that, actual realizations of return volatility are not directly observed like raw returns. A common approach to deal with the fundamental latency of return volatility is to conduct inference regarding volatility through strong parametric assumptions: such that, an autoregressive conditional heteroscedasticity (ARCH) or a stochastic volatility (SV) model estimated with data at daily or lower frequency. However, SV models are beyond the scope of this study and GARCH family models are considered for modeling conditional variance of returns.

These models (including ARCH, GARCH and their many generalizations) have been developed to reflect the so-called stylized facts of financial time series. Their properties, which include tail heaviness, volatility clustering and serial dependence without correlation, cannot be captured with traditional linear time series models. Moreover, the volatility of financial instruments is rarely constant, and usually varies over time. This creates a phenomenon called volatility clustering, where large price movements on one day are followed by similarly large movements on successive days, creating temporal clusters. The GARCH model, which treats volatility as a drift process, is commonly used to capture this behavior. However, Lévy process frameworks are failed to capture effect of volatility clustering (Kumari et al. 2013). Therefore, study of GARCH class models is more prominent. Under this study, gold futures market data is used to model volatility in different manner.

The most important ambassador from the world of commodities is gold except oil. The world market for gold is characterized by worldwide trading. Gold is traded over-the-counter (OTC) worldwide and financial gold products (ETF's, Futures and other derivatives) on a wide variety of organized exchanges and platforms. The world demand of newly mined gold is roughly divided as follows: 50% for jewelry, 40% for investment, and only 10% for industrial purposes (Thompson 2012).

Over past few years, the volatile price of gold has caused great concerns among market participants and researchers. Volatility is a major input in calculating value at risk (VaR) and derivative price, thereby forecasting and modeling gold price volatility have important theoretical and practical implications. Despite the importance of gold as a hedge and a safe haven asset (Baur and Lucey 2010) studies investigating the volatility of gold future market are rare. Tully and Lucey (2007) specify an asymmetric component in APGARCH model but they find that the asymmetry is statistically insignificant. Batten and Lucey (2010) model the volatility of a gold futures market. Moreover, Batten and Lucey (2010) find monetary variables to explain gold volatility. Later, the behavior of gold prices is covered by Lucey, Larkin and O'Connor (2013).

The sources of variability changes are elusive asset returns without knowing why volatility changes. This is the path that will focus under this study. Conditional variance of returns is provided insights into the movement of volatility through time. This study attempts to model and forecast the volatility of gold futures trading at the COMEX during 1990-2014, using various models from the GARCH family. For better capturing the dynamics of gold volatility, GARCH-class models incorporating long memory were employed. Some extreme events like bad weather and financial crisis can cause large changes in gold price, resulting in the fat-tail distribution and asymmetry of price returns. Therefore, different type of error distribution to model excess kurtosis and skewness were considered. Further, different GARCH models are considered to capture volatility asymmetry of gold. Finally, volatilities are forecasted using GARCH-class models and out-of-sample performances of models are evaluated based on different statistical tests.

2. Research Methodology

ARCH models define conditional distribution for returns that are characterized by time-varying conditional variance. ARCH modeling has rapidly become a dominant paradigm when discrete-time models are used to describe the prices of financial assets. These models are the basic econometric tools used to estimate and forecast asset returns volatility. In this section, the succinctly different ARCH models are discussed.

2.1. ARCH model

In a seminal paper, Engle (1982) propose to model time-varying conditional variance with the autoregressive conditional heteroscedasticity (ARCH) processes that use past disturbances to model the variance of the series. The distribution of all the return for period t, conditional on all previous returns, is normal with mean μ and time-varying conditional variance h_t defined by;

$$r_t \mid r_{t-1}, r_{t-2}, \ldots \sim N(\mu, h_t), \quad (1)$$

and

$$h_t = \omega + \alpha(r_{(t-1)} - \mu)^2. \quad (2)$$

The volatility parameters are $\omega > 0$ and $\alpha \geq 0$. The volatility of the returns in period t then depends solely on the previous return. A large positive/negative return in period $t-1$ implies higher than average volatility in the next period when α is positive. Furthermore, returns near the mean level μ imply lower average future volatility.

2.2. GARCH model

Empirical evidence shows that high ARCH order has to be selected in order to catch the dynamic of the conditional variance. The generalized ARCH (GARCH) model of Bollerslev (1986) is an answer to this requirement as it is based on an infinite ARCH specification which reduces the number of estimated parameters from infinity to two. The distribution of all the return for period t, conditional on all previous returns, is defined GARCH(1,1) model as follows:

$$r_t \mid r_{t-1}, r_{t-2}, \ldots \sim N(\mu, h_t), \quad (3)$$

with

$$h_t = \omega + \alpha(r_{(t-1)} - \mu)^2 + \beta h_{t-1}. \quad (4)$$

There are four parameters, namely μ, α, β and ω. The constraints $\omega \geq 0, \alpha \geq 0$ and $\beta \geq 0$ are required to ensure that conditional variance is never negative. Therefore, the standard GARCH(p, q) model expresses the variance at time t and h_t is given by:

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}, \quad (5)$$

where $\varepsilon_{t-i} = r_{t-i} - \mu$ is a residual at time t and μ, α_i, β_j and ω are the parameters to be estimated, q is the number of lags for past variances, and p is the number of lags for past squared residuals. Therefore, GARCH model allows both autoregressive and moving-average components in heteroscedastic variance. It gives a more parsimonious representation of the ARCH model and is much easier to identify and estimate.

2.3. ARFIMA model

To take into account the role of long memory in returns process, autoregressive fractionally integrated moving average (ARFIMA) model is considered by Baillie (1996). The specification of ARFIMA(1,d,1) process can be written as follows:

$$(1-L)^d(1-\phi L)(r_{t-1}-\mu)=(1-\theta L)\varepsilon_t, \tag{6}$$

where μ is the return mean, d is the fractional difference operator capturing long memory, and ϕ are parameters to be estimated and L is the lag operator[1]. This process is stationary when $d<0.5$.

It is worth nothing that both the models (ARCH & GARCH) are captured leptokurtosis and volatility clustering. However, they fail to capture the leverage effect[2] and also long memory in volatility process. To address these problems, various extensions of nonlinear GARCH-class models have been proposed by many authors and these models are discussed in below.

2.4. EGARCH model

The first model to account for such effects was the exponential GARCH (EGARCH) model proposed by Nelson (1991). It uses a logarithmic function to treat asymmetric effects, and EGARCH(p,q) is given by:

$$\ln(h_t)=\omega+\sum_{i=1}^{p}\alpha_i\left(\left|\frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}}\right|-\sqrt{\frac{2}{\pi}}\right)-\sum_{i=1}^{p}\gamma_i\left(\frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}}\right)+\sum_{j=1}^{q}\beta_j\ln(h_{t-j}). \tag{7}$$

As claimed by Nelson (1991), there are no restrictions on parameters in EGARCH.

2.5. APARCH model

Ding (1993) introduce the Asymmetric Power ARCH (APARCH) model. The APARCH(p,q) model can be expressed as:

$$h_t^{\vartheta/2}=\omega+\sum_{i=1}^{p}\left(\alpha_i|\varepsilon_{t-i}|+\gamma_i\varepsilon_{t-i}\right)^{\vartheta}+\sum_{j=1}^{q}\beta_j h_{t-j}^{\vartheta/2}, \tag{8}$$

where $\omega>0, \vartheta\geq 0, \beta_j\geq 0, (j=1,2,...,q),$ $\alpha_i\geq 0$ and $-1<\gamma_i<1, i=1,...,p.$ The effect of good and bad news is captured separately through the two coefficients, α and γ, respectively. This model can capture the leverage effect.

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[1] ARMA(1,1) model is defined by:
$r_t-\mu=\phi(r_{t-1}-\mu)+\varepsilon_t+\theta\varepsilon_{t-1}$ and according to lag operator L, (defined by) $La_t=a_{t-1}$, this equation can be rewritten as: $(1-\phi L)(r_{t-1}-\mu)=(1+\theta L)\varepsilon_t$.

[2] This relates to the tendency of stock returns to be negatively correlated with changes in return volatility.

2.6. TARCH model

Threshold ARCH (TARCH) model (Zakoïan 1994) is given by:

$$h_t^{1/2} = \omega + \sum_{i=1}^{p}\left(\alpha_i \varepsilon_{t-i}^{+} + \gamma_i \left|\varepsilon_{t-i}^{-}\right|\right) + \sum_{j=1}^{q}\beta_j h_{t-j}^{1/2}, \tag{9}$$

where $\varepsilon^{+} = \max(\varepsilon, 0)$ and $\varepsilon^{-} = \min(\varepsilon, 0)$. The effect of good and bad news is captured separately through the two coefficients, α and γ, respectively.

2.7. FIGARCH and FIEGARCH models

Unlike the univariate models mentioned above which are based on the hypothesis that the volatility autocorrelation decays at an exponential rate. Apart from that, Baillie et al. (1996) propose a fractionally integrated GARCH model (FIGARCH) allowing for the hyperbolic rate decaying of autocorrelations. Interestingly, FIGARCH$(1,\rho,1)$ nests a GARCH(1,1) with $\rho = 0$. The FIGARCH$(1,\rho,1)$ model can be written as follows:

$$h_t = \omega + \beta h_{t-1} + \left[1 - (1-\beta L)^{-1}(1-\varphi L)(1-L)^{\rho}\right]\varepsilon_t^2, \tag{10}$$

where $0 \le \rho \le 1$, $\omega > 0$, $\beta, \varphi < 1$, ρ is the fractional integration parameter and L is the lag operator.

The parameter ρ characterizes the long memory property in volatility. The advantage of the FIGARCH process is that for $0 < \rho < 1$, it is sufficiently flexible to allow for intermediate ranges of persistence. If $\rho = 0$ volatility shocks decay with a geometric rate and if $\rho = 1$, volatility shocks have complete integrated persistence.

Bollerslev and Mikkelsen (1996) proposed a flexible fractionally integrated EGARCH (FIEGARCH) model which allows for both long memory in volatility process and the asymmetric effect. The FIGARCH$(1,\rho,1)$ model can be described as:

$$\ln(h_t) = \omega + (1-\beta L)^{-1}(1+\varphi L)(1-L)^{-\rho}\left[\gamma z_{t-1} + \alpha\left(|z_{t-1}| - E\left[|z_{t-1}|\right]\right)\right] \tag{11}$$

where, $\frac{\varepsilon_t^2}{z_t^2} = h_t$. This FIEGARCH model nests the conventional EGARCH for $\rho = 0$, and the IEGARCH model for $\rho = 1$. At a slow hyperbolic rate of decay, the effect of a shock to the forecast of $\ln(h_t)$ is dissipated for $0 < \rho < 1$.

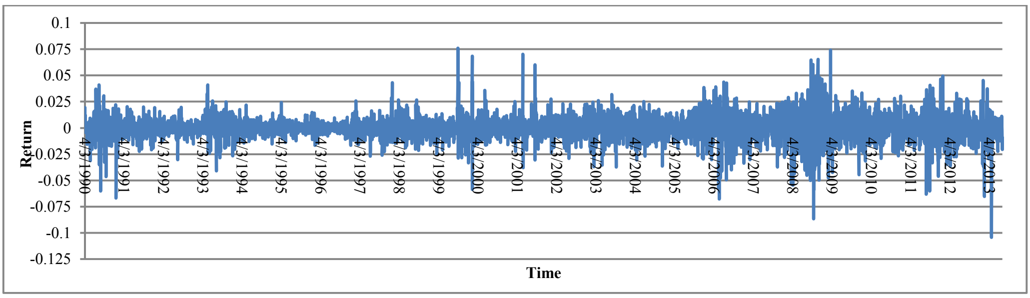
In summary, linear GARCH-class models with five nonlinear GARCH-class models (EGARCH, APARCH, TARCH, FIGARCH, FIEGARCH) are employed to model and forecast gold market volatility under different error distribution as mention below. Most of the time, financial time-series often exhibits non-normality patterns, i.e. skewness and excess kurtosis. GARCH models do not always fully embrace this property of high frequency financial time-series. To overcome this drawback Bollerslev (1986) and Beine et al. (2002) have used the Student's t-distribution. Similarly to capture skewness, Liu and Brorsen (1995) have used an asymmetric stable density. To model both skewness and kurtosis Fernandez and Steel (1998) used the skewed Student's t-distribution which was later extended to the GARCH framework by Lambert and Laurent (2001). To improve the fit of the GARCH and EGARCH models into international equity markets, Harris et al. (2004) used the skewed generalized Student's t-distribution to capture the skewness and leverage effects of daily returns. In addition to the Normal distribution, Student t-distribution and skewed Student t-distributions are consider to reduce the excess kurtosis and skewness of time-series data (Kumari 2014) with respective their log-likelihood functions.

3. Results and Discussions

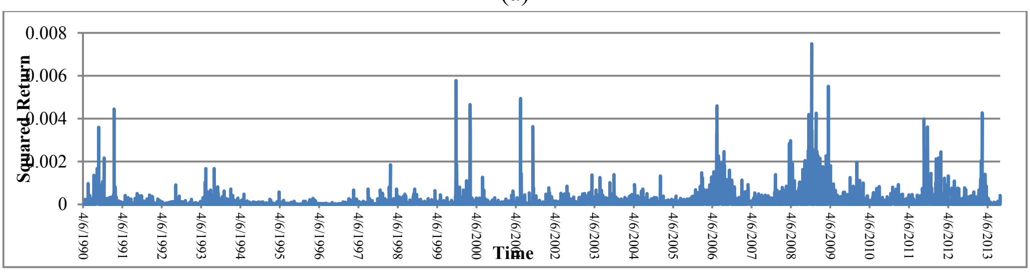
3.1. Data and preliminary analysis

Gold futures prices traded on the COMEX are considered in this study. In order to calculate logarithm of these returns, the daily prices of these futures data are used (Kumari, 2014) and which are obtained from the Bloomberg database. The prices are taken as the last trading daily price of a given day. The data on futures prices cover 6373 observations from January 1990 to June 2014. The data ranging from January 1990 to June 2013 are used for modeling purposes, i.e., a total of 6135 observations, which is sufficient for modeling daily returns. The remaining data from July 2013 to June 2014 (238 observations) are treated as an out-of-sample period in order to assess the forecasts made.

Daily returns can be calculated using this formula $r_t = \ln S_t - \ln S_{t-1}$, where, r_t is daily returns and S_t is gold price at time t). Squared returns are considered as the proxy of volatilities. Both data series are plotted in the Figure 1. The figures demonstrate the association between returns and volatility and the occurrence of extreme returns and high volatility. As an example, during the sample period, four episodes of increased volatility and extreme return shocks can be identified. The first two episodes (1999 and 2001) are relatively short compared to the second set of episodes (middle of 2005 and 2007). The highest episode of increased volatility can be linked to the global financial crisis of 2007 and 2008.



(a)



(b)

Figure 1 Daily returns of gold (in US$)-(a) and the squared return as a proxy for volatility-(b)

Table 1 Descriptive statistics of price returns of gold futures

Mean (%)	Maximum	Minimum	Standard Deviation (%)	Skewness	Excess Kurtosis	Jarque-Bera
0.0276	0.0971	-0.0865	3.5434	0.1045	9.5721	10,694.2*
Q(1)	Q(10)	Q2(1)	Q2(10)	ADF	PP	ARCH(2)
12.963*	37.521*	85.716*	102.841	-35.109*	-36.542*	86.5*

Note: * Denote rejection of the null hypothesis at the 1% significance level.

This period shows high volatility levels and extreme realizations of positive and negative returns with positive returns being more frequent than negative returns. The proxy for the volatility of gold displays clusters of high and low volatility. Since dynamics of gold prices are volatile; therefore, modeling and forecasting price volatility are of great importance for market participants.

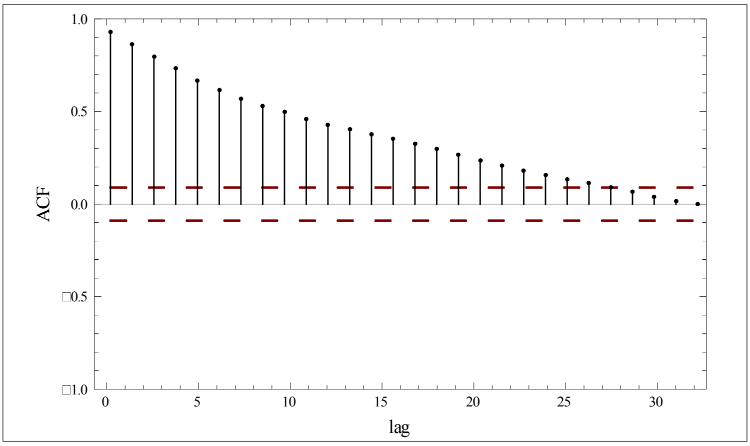
Descriptive statistics of futures price returns are reported in Table 1 with the results of unit root tests for return series based on the Augment Dickey and Fuller (ADF) and Phillips and Perron (PP) methods. The Jarque and Bera statistics show the rejections of the null hypothesis of normal distribution at the 1% significance level implying that fat-tail distribution as evidenced by the positive skewness and excess kurtosis. Thus, it is necessary for us to model volatilities by taking the stylized fact of fat-tail distribution into account. By performing the further test on stationary of the return series using the Augmented Dick Fuller test (ADF) (Dickey and Fuller 1979) and Phillips Perron (Phillips Perron 1988) (PP) unit root tests and results are reported in also Table 1. Both tests indicate that the null hypothesis of a unit root is rejected. That means, the return series of gold futures prices can be considered to be stationary. The ADF test is set to a lag length 0 using the Schwarz Information Criterion (SIC) and the PP test is conducted using the Bartlett Kernel spectral estimation method. Moreover, the Ljung and Box's Q statistics (Ding et al. 1993) consistently show the rejections of no autocorrelations up to the first and tenth orders implying the existence of serial correlations in returns and squared returns and strong ARCH effects.

3.2. In-sample performance

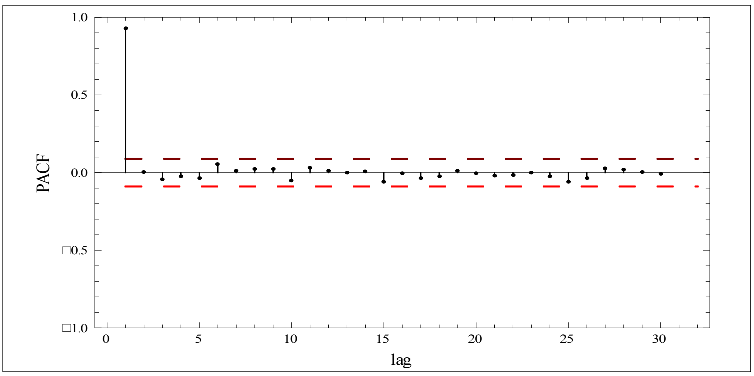
In order to model the volatility of the returns the mean equation of returns[3] is required. The plot of ACF and plot of PACF in Figure 2 reveal that autoregressive moving average, ARMA(p,q) is more suitable to model the mean returns. After considering several parsimonious models, ARMA(1,1) is found to be a significant model for the mean equation; with a Wald statistic of 254.21 and significant t-values for the coefficients. Moreover, higher order terms in ARMA model are insignificant. However, there is a little evidence to present long memory effect in the return series due to slow decay of ACF plot in Figure 2-(a), (The plot of autocorrelations looks little closer to hyperbolic than exponential). To account this fact, ARFIMA model is considered as a mean equation of returns apart from ARMA model and results are reported in Table 2. The parameter estimate of d in the mean equation is

[3] The return for today will depend on returns in previous periods (AR component) and the surprise terms in previous periods (MA component). Plotting the autocorrelation and partial autocorrelation of the returns series can help determine the order of the mean equation.

insignificant. This is implied the absence of significant long memory in gold returns. All other parameters (AR and MA) are statistically significant. The residuals of the mean equation indicate the absence of autocorrelation through the Q-statistic (Figure 3 (a)-ACF and Figure 3 (b)-PACF). Moreover, Engle (1982) ARCH-Lagrange Multiplier4 (LM) test provides a Chi-squared value of 51.24, confirming the presence of ARCH effect. Thus there is a need to model this conditional variance using the ARCH class models.



(a) ACF of returns



(b) PACF of returns

Figure 2 Plot of ACF and PACF of return series

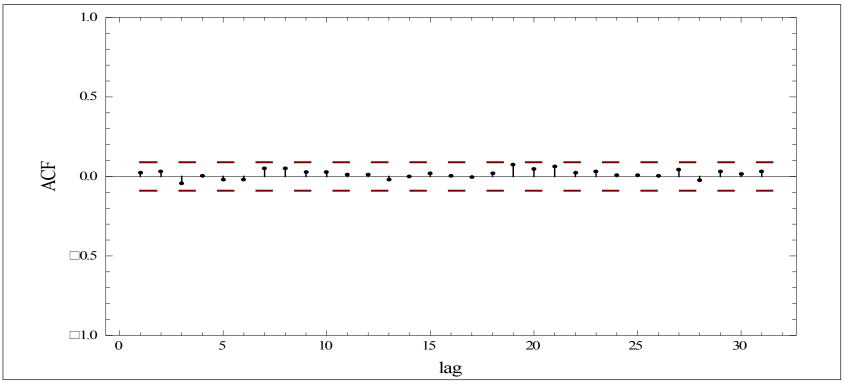
Note: 95% Confidence bands [Standard Error = 1/sqrt (no of observations)]

[4] H_0 : No ARCH effects vs. H_1: ARCH(p) disturbance

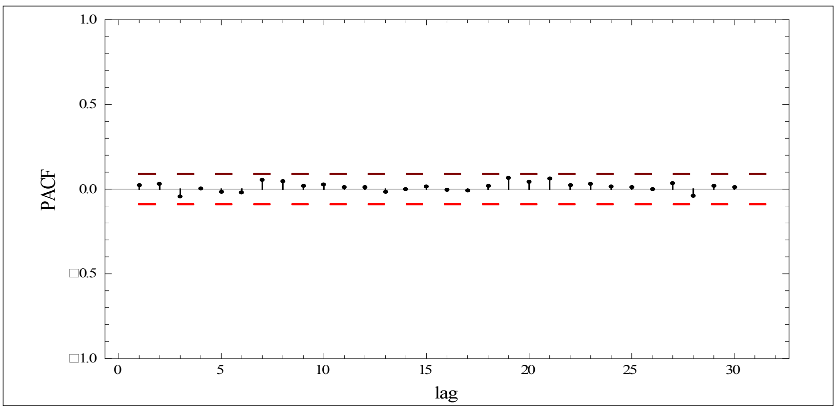
Table 2 Estimated results of ARFIMA(1,d,1) model

Mean Equation: $(1-L)^d\left((r_t-\mu)-\phi(r_{t-1}-\mu)\right)=\varepsilon_t+\theta\varepsilon_{t-1}$

Parameters	μ	θ	ϕ	d
Coefficients	-0.000075	0.3381	-0.8439	0.3453
Standard Error	0.00001	0.51536	0.42481	0.18044
p-value	0.1087	0.0325	0.0254	0.1152



(a) ACF of returns



(b) PACF of returns

Figure 3 Plot of ACF and PACF of residuals of mean equation

Note: 95% Confidence bands [Standard Error = 1/sqrt(no of observations)]

Table 3 Estimation results of ARMA(1,1)-GARCH(1,1) model

Mean Equation: $(r_t - \mu) = \phi(r_{t-1} - \mu) + \varepsilon_t + \theta\varepsilon_{t-1}$

Variance Equation: $h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$

Parameters	Error term distribution		
	Normal	Student-t	Skewed-t
μ	-0.000075 (0.00097)	0.000085 (0.00081)	0.000082 (0.00089)
ϕ	-0.5028* (0.20105)	-0.1021* (0.28871)	-0.1147* (0.24720)
θ	0.4875* (0.20307)	0.0922* (0.28938)	0.0755* (0.24788)
ω	0.0000042*** (0.000059)	0.0000026** (0.000094)	0.0000031*** (0.000011)
α	0.0620*** (0.0025)	0.0672*** (0.0069)	0.0632*** (0.0063)
β	0.9385*** (0.0023)	0.9402*** (0.0051)	0.9377*** (0.0056)
ν	-	3.85*** (0.2342)	3.94*** (0.2761)
δ		-	-0.0245* (0.0287)
R-squared (%)	7.31	12.29	13.77
Q(10)	0.458 [0.725]	2.354 [0.851]	2.914 [0.873]
ARCH(10)	0.9153 [0.531]	0.8217 [0.621]	0.8218 [0.619]
AIC	-6.54	-6.66	-6.68
Log-likelihood	19432.1	19781.9	19831.2

Note: Standard errors are given in parentheses and p-values of the statistics are reported in square brackets.

Q(10) is the Ljung and Box (1978) Q-statistics of orders 10 computed on the squared standardized residuals. ARCH(10) is the ARCH-Lagrange Multiplier (LM) test statistics of orders 10. AIC is the Akaike Information value.

*, ** and ***, denote the 10%, 5% and 1% level of significance, respectively.

Estimation results of GARCH (1,1) model is reported in Table 3 with the ARMA (1,1) as the underlying mean equation for futures price returns of gold. This model is estimated by approximate quasi-maximum likelihood under normal, Student-t and skewed Student-t errors. Both the coefficient of the mean and variance equation is statistically significant. A value of β for past variance in GARCH model implies that the shock of past volatility has a persistent effect on future volatility. The sum of the two coefficients $(\alpha + \beta)$ is a succinct measure of the persistence of variance, and that its value is close to 1 implies that there is significant persistence in volatility.

The regression R-squared is low. It implies that other factors drive changes in price other than the AR and MA coefficients. This is consistent with trading in the gold futures market. Q-statistic of standardized residuals reveals that the errors are white noise. This means that higher-order GARCH models are not required. Therefore, that the GARCH(1,1) model is able to appropriately capture the GARCH effects. It is further implied by the ARCH-LM test.

The Student-t and skewed-t distributions clearly outperformed than Gaussian. Indeed, the log-likelihood function strongly increases when using the Student-t to skewed-t, while AIC values are decreases. Therefore, Skewed Student-t gives better results than the symmetric Student-t when modeling the gold future return. The addition of two asymmetric parameters (asymmetric GARCH and asymmetric distribution) may therefore be necessary. Hence, GARCH-class models are considered to produce better results.

Table 4 Estimated statistics for models comparison under skewed-t distribution

Parameters	Estimated Statistics					
	GARCH	EGARCH	APARCH	TARCH	FIGARCH	FIEGARCH
μ	0.000082 (0.00089)	0.000210** (0.00094)	0.00014 (0.00098)	0.000123 (0.000096)	0.0021 (0.00501)	0.00278** (0.0011)
ϕ	-0.1147* (0.24720)	-0.4025** (0.18323)	-0.4842** (0.19770)	-0.5029** (0.2071)	-0.5449* (0.30069)	-0.4529** (0.1385
θ	0.0755* (0.24788)	0.3792** (0.18501)	0.46742** (0.19964)	0.4867** (0.2092)	0.5411* (0.30152)	0.4221* (0.1411)
ω	0.000031** (0.000011)	-0.1801*** (0.01032)	0.00026* (0.00011)	0.00042** (0.00005)	0.00044** (0.000017)	0.0021 (0.0049)
α	0.0612*** (0.0063)	0.0446*** (0.0031)	0.05950** (0.0032)	0.0855** (0.00345)	-	0.0574** (0.0049)
β	0.9377*** (0.0056)	0.9002*** (0.00091)	0.9432*** (0.0023)	0.94130*** (0.0021)	0.8623*** (0.0147)	0.9498*** (0.0003)
γ	-	0.0827*** (0.01032)	0.0595* (0.0032)	-0.0548* (0.00404)	-	0.1190* (0.0143)
ϑ	-	-	1.345* (0.5471)	-	-	-
φ	-	-	-	-	0.315* (2.188)	-0.157 (0.298)
ρ	-	-	-	-	0.314* (2.141)	0.421* (2.991)
ν	3.94*** (0.2761)	3.88* (0.2383)	3.92* (0.2436)	3.92** (0.2454)	3.93** (0.2341)	4.14*** (0.1868)
δ	-0.054* (0.0287)	-0.067* (0.0184)	-0.034 (0.0247)	-0.028 (0.0224)	-0.058* (0.0158)	-0.068* (0.0098)
R-squared (%)	13.77	14.97	12.82	13.19	13.11	14.77
Q(10)	0.914 [0.873]	0.458 [0.725]	0.358 [0.544]	0.317 [0.891]	0.495 [0.752]	0.471 [0.711]
RCH(10)	0.8218 [0.619]	0.9153 [0.531]	0.8117 [0.604]	0.3874 [0.754]	0.9137 [0.511]	0.8446 [0.485]
IC	-6.68	-6.97	-6.58	-6.66	-6.65	-6.81
Log-likelihood	19831.2	20998.1	19801.2	19791.81	19780.0	20241.8

Note: Standard errors are given in parentheses and p-values of the statistics are reported in square brackets.

Q(10) is the Ljung and Box (1978) Q-statistics of orders 10 computed on the squared standardized residuals. ARCH(10) is the ARCH-Lagrange Multiplier (LM) test statistics of orders 10. AIC is the Akaike Information value.

*, ** and ***, denote the 10%, 5% and 1% level of significance, respectively.

The results of the estimated parameters and diagnostic tests are listed in Table 4. The use of asymmetric GARCH models seems to be justified. All asymmetric coefficients are significant.In EGARCH model, the asymmetric coefficient γ is significant, implying the present of volatility asymmetries in futures volatilities, also confirmed by the estimates of the coefficients γ in other models. The estimates of $(\alpha+\beta)$ of GARCH(1,1) and $\left(\alpha+\beta+\frac{\gamma}{2}\right)$ of EGARCH are less than unity, satisfying the conditions for existence of the second moment (Ling and McAleer 2003). In addition, the coefficient ϑ in APARCH model is significant at 10% level. There is some evidence to present leverage effect too. Furthermore, the long memory parameter ρ in both FIGARCH and FIEGARCH models are significantly different from zero at least at the 10% significance level, implying the existence of long memory in volatilities.

The values of log-likelihood are close to each other across different models. However, the Log-likelihood value of EGARCH model is slightly larger than those from other models and followed by FIEGARCH model. For futures price returns, Ljung and Box's Q statistics cannot significantly reject the null hypothesis of no serial correlations in squared standardized residuals. This evidence indicates that these GARCH-class models can capture the dynamics of gold price volatility well, also as confirmed by the results of Engle's ARCH test (Bollerslev and Mikkelsen 1996). Overall in-sample-fit, the EGARCH model is performed little better than FIEGARCH model. Further, out-of-sample will be considered to find the better results among selected models.

3.3. Forecasting futures price volatilities (out-of-sample)

The forecasting performance of GARCH-class models has been comprehensively discussed by Poon and Granger in 2003. Compared with the in-sample performance, the out-of-sample performance is important because market participants are more concerned about how well they can do by employing these models. Moreover, in-sample performance provides history performance only. Thus, we forecast gold price volatilities using GARCH-class models and compare their out-of-sample performances.

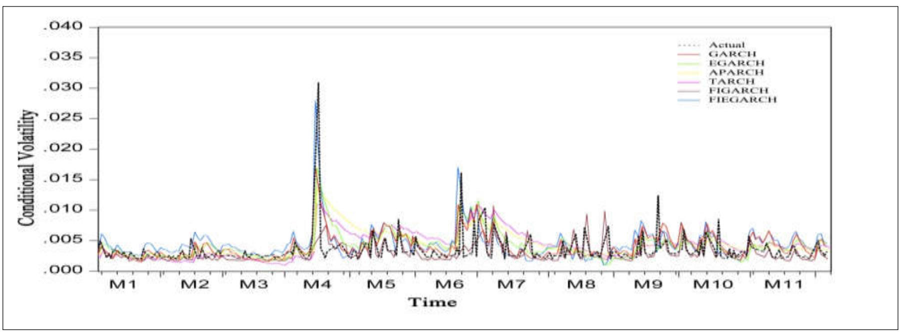


Figure 4 Volatility forecasts of futures prices based on GARCH-class models with Skew t-distributions from July, 2013 to June, 2014

Figure 4 shows the volatility forecasts of futures prices of gold based on GARCH-class models with skew-t distribution. These models provide perfect prediction of volatilities in some period, whereas present the over-prediction of volatilities during the period of some large fluctuations. Therefore, GARCH-class models can lead to the under-predictions. Most of the GARCH-class models show the similar volatility pattern as in the actual volatility series.

To access the performance of the considered models in forecasting the conditional variance, variety of measures are employed, such as mean absolute error (MAE), mean squared error (MSE)5, R2LOG statistics, QLIKE statistic and the Theil's inequality coefficient (TIC);

$$MAE = n^{-1}\sum_{t=1}^{n}\left|\sigma_t^2 - \hat{\sigma}_t^2\right|,$$

$$MSE = n^{-1}\sum_{t=1}^{n}\left(\sigma_t^2 - \hat{\sigma}_t^2\right)^2,$$

$$R^2LOG = n^{-1}\sum_{t=1}^{n}\left[\ln\left(\frac{\sigma_t^2}{\hat{\sigma}_t^2}\right)\right]^2,$$

$$LIKE = n^{-1}\sum_{t=1}^{n}\left[\ln\left(\sigma_t^2\right) + \frac{\sigma_t^2}{\hat{\sigma}_t^2}\right]$$

and

$$TIC = \frac{\sqrt{\frac{1}{h+1}\sum_{t=1}^{n}\left(\hat{Y}_t - Y_t\right)^2}}{\sqrt{\frac{1}{h+1}\sum_{t=1}^{n}\left(\hat{Y}_t\right)^2} - \sqrt{\frac{1}{h+1}\sum_{t=1}^{n}\left(Y_t\right)^2}}, \quad (12)$$

where n is the number of forecasts, σ_t^2 and $\hat{\sigma}_t^2$ are the actual volatility and the volatility forecasts obtained from GARCH-class models respectively.

Table 5 Comparisons of out-of-sample forecasting performance of GARCH-class models with skewed-t distribution

Models	Model Selection Criteria				
	MAE	MSE	R2LOG	QLIKE	TIC
GARCH	5.123	85.69	1.251	1.118	0.344
	[0.322]	[0.558]	[0.425]	[0.547]	[0.558]
EGARCH	4.925	80.41	1.021	0.988	0.229
	[0.875]	[0.982]	[0.654]	[0.958]	[0.988]
APARCH	6.654**	99.14**	1.896*	1.874***	1.000***
	[0.032]	[0.045]	[0.058]	[0.009]	[0.000]
TARCH	5.847*	101.21***	1.947***	1.647**	0.854***
	[0.098]	[0.003]	[0.009]	[0.021]	[0.009]
FIGARCH	6.194**	95.87*	1.755*	1.541*	0.554*
	[0.048]	[0.099]	[0.061]	[0.054]	[0.087]
FIEGARCH	5.338	88.33	1.541*	0.991	0.225
	[0.117]	[0.397]	[0.098]	[0.998]	[1.000]

Note: Out-of-sample forecast results for the period July, 2013 to June, 2014. Results are reported for examined models and the considered performance. p-values of the statistics are reported in square brackets and the highest p-values are in bold face.

*, ** and ***, denote the 10%, 5% and 1% level of significance respectively.

5 Andersen, T. G., Bollerslev, T., and Lange, S. in 1999, used the MSE and MAE statistics.

Table 5 reports the results for an out-of-sample analysis of the all models by comparing one-step-ahead volatility from July, 2013 to June, 2014 under five different criteria. For the gold futures, the results support the use of the asymmetric EGARCH model. According to most measures in the variance equation, the EGARCH model outperforms6 than FIEGARCH model with the assumption of skewed-t distributed innovations. According to the QLIKE and TIC statistic, FIEGARCH model is performed better than the other models while other statistics give best results except R2LOG. That means; there is some evidence to present long memory effect in volatilities. The GARCH model provides satisfactory results while APARCH and TARCH models provide the poorest forecasts.

4. Conclusions

The recent global financial crisis has highlighted the need for financial institutions to find and implement appropriate models for risk measurement. There was a particular interest of investors to increase their positions in the gold market as the risk in equity and bond markets was increasing. This study evaluates the effectiveness of various volatility models with respect to modeling and forecasting market risk in the gold future market.

The gold futures volatility modeled and forecasted by using GARCH-class models with long memory and fat-tail distributions, by considering ARMA model as the conditional returns. The results reveal that ARMA(1,1) model provides best results for the conditional returns. Among the linear and non-linear GARCH-class models, EGARCH and FIEGARCH models are provided best results for in-sampling forecasting. Moreover, EGARCH model gives bit of higher performance than FIEGARCH model under model diagnostic tests.

After that, futures price volatilities of gold are forecasted using linear and nonlinear GARCH-class models. Based on the model selection criteria, EGARCH and FIEGARCH models are superior to other models in the sense of forecasting accuracy. More importantly, the evidence indicates that long memory effect is significant. In addition, the simple linear GARCH-class model provides high accuracy predictions than other non-linear models (APARCH, TARCH and FIGARCH). Thus, for gold retailers who are frequently exposed to the risk of the future market, the simple linear GARCH-class models are the better choices of risk management. For speculations and noise traders in futures market, both linear and nonlinear models should be taken into account. Furthermore, the forecasting accuracy of GARCH-class models with different distributions of innovations is compared. Among them GARCH-class models with skew t-distribution model is outperform those with normal distribution. By considering the all selection criteria GARCH-class models with skewed-t distribution is best model for future forecasting. Thus, market participants should not neglect the stylized fact of fat-tail distribution when they perform risk management.

[6] The highest p-values are reported under MAE, MSE and R^2LOG while p-values of QLIKE and TIC criteria are also high.

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